

# Achieving Optimum Value of $k$ in a $K$ -fold Multicast Network with Buffer using $M/M/n/n+q/N$ Traffic Model

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## Abstract

*Multicast network is widely used for effective communication, transmission and performance optimizations of a network. In this paper, a new model has been developed to determine a suitable value of the fold  $k$  of a  $k$ -fold multicast network under different traffic loads under Poisson traffic with finite queue at each node. We have derived stationary distribution for the network states and then derived expressions for the network throughput and the blocking probability of the network. It has been found in this research work that the network throughput increases very fast as we increase the fold number. However, at a certain value of the fold, the blocking probability ceases to increase and it remains constant. We have also observed that as the offered traffic is increased, the throughput also increases. Moreover, the system parameter  $k$  is increased, the blocking probability decreases. However, after an optimum value of  $k$ , the blocking probability remains constant for a particular value of the offered traffic. In fact, in this paper, by evaluating the performance of a  $k$ -fold multicast network, our developed model improves the performance of a multicast network.*

## Keywords

K-fold network, Kendal's notation, Markov chain, Multicasting, Throughput, Traffic Theory

## 1. INTRODUCTION

Multicast involves transmitting information from a single source to multiple destinations. This is an important requirement for high-performance communications networks. Multicast communication is one of the most important collective communication operations and is highly

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demanding in broad-band integrated services network (BISDN) and in communication-intensive applications in parallel and distributed computing systems, such as distributed database updates and cache coherence protocols. It is projected that multicast will also be increasingly used to support various other interactive applications such as multimedia, teleconferencing, web servers and electronic commerce on the Internet. [1] Many of these applications require predictable communications performance, such as guaranteed multicast latency and bandwidth, called quality of service (QoS) in addition to multicast capability. The QoS guarantees and the non-uniform nature of multicast traffic make the problem of the analysis of multicast communication is very challenging.

However, to provide a quantitative basis for the network designers, determining an optimum value of the system parameter  $k$  (the fold number) is essential. Keeping this view in mind, we have, in this research work, developed an analytical model to determine the suitable value of  $k$  under different traffic loads for a  $k$ -fold multicast network under Poisson traffic with finite buffers or queue at each node. [2] We have derived stationary distribution for the network states and then derived expressions for the network throughput and the blocking probability of the network. Moreover, in this paper we have shown the adjustable parameter  $K$  with finite users using Markovian model  $M/M/n/n+q/N$ .

## 2. LITERATURE REVIEW

### A. Background

Multicasting is a technical term which is used as a networking technique of delivering messages and information to a group simultaneously from the source. A typical multicasting service is shown in the Figure 1.



Figure 1: Basic multicast service

In a  $K$ -fold multicast network, fold number indicates the number of request coming from different sources to a particular destination. On the other hand, finite queue is a data set shared by program processes which acts as a buffer for data in multicast network. In this research work, we have developed a model to help network engineers to design an effective multicast network. To do this, it is necessary to get the optimum value of  $k$  in a  $K$ -fold multicast network. By implementing the

optimum value of the system parameter  $k$  in  $K$ -fold multicast network, we improved the network performance by optimizing network throughput, where throughput is the number of messages successfully delivered per unit time. In this paper, the term “throughput” has been used to measure from the arrival of the first bit of data at the receiver. [3]

As mentioned earlier, the primary target of this research work was to evaluate the performance of  $K$ -fold multicast network by using traffic model. For this reason, throughout this paper, we make following assumptions on the multicast traffic we consider.

- The probability of a destination node being involved in an incoming multicast connection request is independent of other destination nodes.
- Multicast connection requests at different source nodes are independent to each other.
- Holding time of each multicast connection is exponentially distributed with parameter and is independent to each other.
- Multicast connection requests arrive at each source node according to a Poisson process with intensity and are independent to each other. [4]

### *B. Previous Researches*

This research work is basically the extension of previous works of Zhenghao Zhang et.al [3] [5] who evaluated the performance of  $k$  fold network but they did not use buffers. After that Asfara R. Towfiq et.al [6] again checked the performance of  $K$ -fold network with a new look. They show the optimization of  $K$ -fold multicast network with buffers but for infinite users. Here, we have used finite users to evaluate performance of  $k$ -fold multicast network by using Markovian model.

For this approach, a destination node may be simultaneously involved in two multicast connections. Such connections will be blocked in a network which is designed to be nonblocking or rearrange able for only multicast assignments. Specifically, the network can realize multiple multicast assignments in a single pass with a guaranteed latency.

## **3. TRAFFIC THEORY AND K-FOLD MULTICAST NETWORK**

### *C. Basic Traffic theory and Markov Chain*

Traffic Theory describes the key models of traffic flow and associated traffic phenomena such as conflicts in traffic, congestion control and effective management of traffic.

In this paper, we have derived stationary distribution of the  $K$ - fold network from which we can obtain network throughput and the blocking probability. We assume the Markovian  $M/M/n/n+q/N$  model which is shown in the Figure 2.



n: Number of users  
 K=n+q: Sum of channels and queue  
 q: Length of queue

#### 4. MATHEMATICAL ANALYSIS

Let us consider that there are  $j$  multicast connection requests, and let  $p_{\text{deg}}(j, m)$  be the probability that a destination node is the destination of exactly  $m$  of the multicast connection requests; or we can say that a destination node is of degree  $m$  under these  $j$  multicast connection requests. The probability that any multicast connection request chooses this destination node is  $\theta$  and is independent of other multicast connections. Thus, we have

$$p_{\text{deg}}(j, m) = \binom{j}{m} \theta^m (1-\theta)^{j-m}, \quad m \in \{0, 1, \dots, j\} \quad (1)$$

which is a binomial random variable. We assume that each destination node has the same distribution given by (1).

Furthermore, we assume that whether a destination node is chosen by a multicast connection is independent of other destination nodes. Thus, in addition to having the same distributions, the degrees of the destination nodes are also independent of each other. That is why, they are a group of independent, identically distributed (i.i.d.) random variables. [9]

##### F. Mathematical Analysis of network throughput

Let  $P_{\text{mc}}(j)$  be the probability that  $j$  multicast connection requests are mutually compatible (*m.c.*) in a  $k$ -fold multicast network. We note that a set of multicast connection requests are *m.c.* when none of the destination nodes has a degree more than  $k$  when realized simultaneously in the network. From (1), it is obvious that the probability of a destination node having a degree less than or equal to  $k$  is  $\sum_{m=0}^k p_{\text{deg}}(j, m)$  for  $j > k$ , and 1 for  $j \leq k$ , because when  $j \leq k$ , no destination node can have a degree more than  $k$ . Since the degrees of destination nodes are independent of each other, we have

$$P_{\text{mc}}(j) = \begin{cases} \left( \sum_{m=0}^k p_{\text{deg}}(j, m) \right)^n, & j > k \\ 1, & \text{otherwise.} \end{cases} \quad (2)$$

Now, let us consider that a new multicast connection request arrives when  $j$  multicast connections are already in the network. If this new connection can be realized along with those ongoing connections, we say that it can *join* the ongoing connections. Let  $P_{\text{jn}}(j)$  be the probability that a new multicast connection can join  $j$  ongoing connections. It can be shown that

$$P_{\text{jn}}(j) = \frac{P_{\text{mc}}(j+1)}{P_{\text{mc}}(j)}. \quad (3)$$

\By solving the Markov chain of Figure 2, the stationary states are found to have the probabilities

$$P_r = \binom{N}{r} \rho^r \prod_{x=0}^{r-1} P_{jn}(x) P_0, \quad 0 \leq r \leq n$$

Total number of times the network departs from state i due to the arrival of a successful connection request is,

$$TP_j(n+q-j)\lambda P_{jn}(j)$$

This is also the total numbers of successful connection requests among at the network when the network is in state  $j(j \in \{0,1,\dots,N\})$  during  $[0,T]$  Therefore the total number of successful connection requests carried by the network during  $[0,T]$  is obtained by summing average,

$$N_{succ} = T\lambda \sum_{j=0}^{n+q} P_j(n+q-j)P_{jn}(j)$$

Therefore the network Throughput is,

$$T_H = \frac{N_{succ}}{T} = \lambda \sum_{j=0}^{n+q} p_j p_{jn}(j)(n+q-j) \quad (4)$$

#### G. Mathematical Analysis of blocking probability

The total number of connection requests arriving at the network during  $[0, T]$

$$N_{total} = (n+q)\lambda T$$

Thus the Blocking Probability,

$$P_B = \frac{N_{bl}}{N_{total}} = \left[ 1 - \frac{1}{n+q} \sum_{j=0}^{n+q} (n+q-j)P_{jn}(j)P_j \right] \quad (5)$$

#### H. Mathematical Analysis of Probability of delay

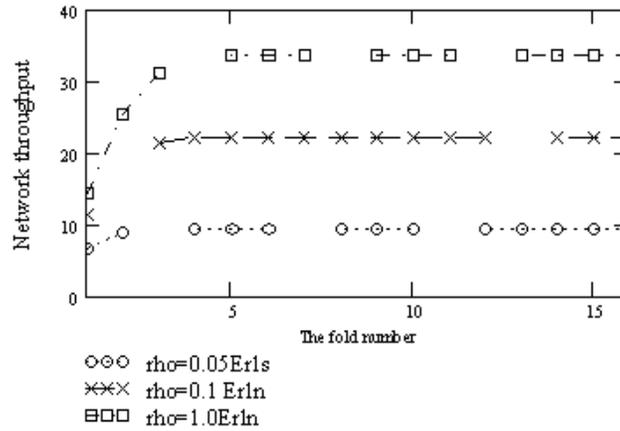
The Probability of Delay is,

$$\begin{aligned} P_D &= \sum_{s=1}^q P_{n+s} \\ &= \sum_{s=1}^q (n+s)! \frac{P^n}{n!} \left( \frac{P}{n} \right)^s \binom{N}{n+s} \prod_{l=0}^{n+s-1} P_{jn}(l) P_0 \end{aligned} \quad (6)$$

## 5. RESULTS AND DISCUSSIONS

For numerical appreciation of our results, we have plotted in Figs. (3), (4) and (5), the throughput and the blocking probability as a function of the fold number  $k$ . [10]

It is seen from Figure 3 that if the fold of the network is increased, network throughput increases very fast in the lower values of the system parameter  $k$ , in our study up to  $k=5$ ; beyond this value of  $k$ , the network throughput is almost constant with respect to the system parameter  $k$  for particular offered traffic. We also observe that as the offered traffic is increased, the throughput



also increases. [11]

Figure 3: Network throughput as a function of the fold number under different offered traffic (N=50, n=14, q= 5,  $\theta = .31$ )

Figure 4 shows the variation of the blocking probability with respect to the fold  $k$ . It is seen from this figure that as the system parameter  $k$  increases, the blocking probability decreases. However, after an optimum value of  $k$ , in our present study it is  $\sim 5$ , the blocking probability remains constant for particular value of the offered traffic.

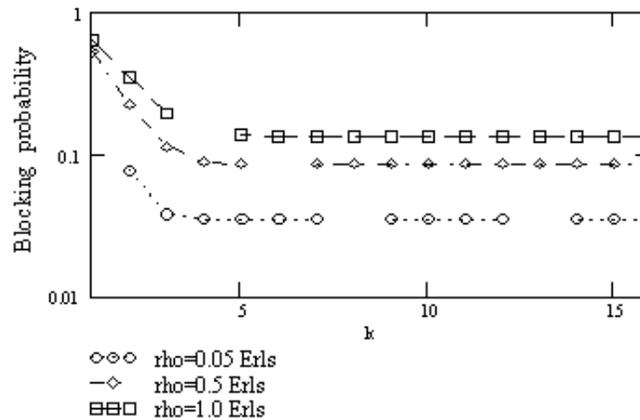


Figure 4: Blocking Probability as a function of the fold number under different offered traffic (N=50, n=14, q=5,  $\theta = .31$ )

Figure 5 shows, the variation of the probability of delayed service with respect to the fold number  $k$ . It is observed that the probability of delay is almost negligible for lower values of the fold number  $k$ , whereas, it is suddenly increases as the fold number approaches the optimum value  $k \sim 5$ . However, after a certain value of  $k$ , the probability of delay becomes constant. [12]

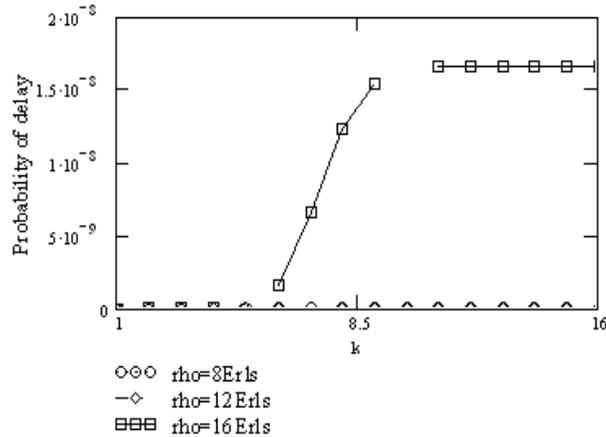


Figure 5: Probability of delay as a function of the fold number under different offered traffic ( $N=50$ ,  $n=14$ ,  $q=5$ ,  $\theta = .31$ )

## 6. CONCLUSIONS

Determining the system parameter  $k$  (the fold number) and finding its optimum value is must to design an effective network. To keep this view in mind, we have developed a systematical model to determine an optimum value for a  $k$ -fold multicast network under Poisson traffic with finite queue at each node. We have derived stationary distribution for the network states and then derived expressions for the network throughput and the blocking probability of the network. It has been found in this study that the network throughput increases very fast as we increase the fold number. However, at a certain value of the fold, the blocking probability ceases to increase and it remains constant. We have also observed that as the offered traffic is increased, the throughput also increases.

In addition, it has been observed that the blocking probability decreased proportionately based on the traffic when the system parameter  $k$  in a  $K$ -fold multicast network is increased. However, after a suitable value of  $k$ , which is approximately 5 based on our research in this paper, the blocking probability remains constant for specific value of the offered traffic in multicast network.

Note that although  $K$ -fold multicast assignments can be realized by simply stacking  $k$  copies of one fold network together, the  $k$ -fold network designed in has a much lower hardware cost. In fact, the cost of the former is about  $3 \cdot k$  times of a  $k$ -fold network for any  $k$ . Thus, a  $k$ -fold network is a cost effective choice to provide better QoS functions in supporting arbitrary multicast communication.

To sum up, this model could help to find out more suitable and appropriate value of the system parameter  $k$  in a  $K$ -fold multicast network to increase throughput of the network. Moreover, this model can be used for designing an effective ubiquitous network in future.

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## REFERENCES

- [1] Md. Syeful Islam, Md. Rezaul Rahman, Anupam Roy, Md. Imdadul Islam, and M. R. Amin, "Performance Evaluation of a Finite Queue Switching Under Two-Dimensional M/G/1(m) Traffic", *International Journal of Imaging Systems (IJIS)*, (Korea), vol. 7, no. 4, pp. 679-690, 2011.
- [2] Anupam Roy, Md. Imdadul Islam, and M. R. Amin, "MMPP+M/D/1 Traffic Model in Video-Data Integrated Service under ATM System", *International J. Engineering and Technology (Singapore)*, vol. 3, no. 6, pp. 615-620, 2011
- [3] Zhenghao Zhang and Y. Yang, "Performance analysis of k-fold multicast networks", *IEEE Trans. Commun.*, vol. 53, pp. 308-314, Feb. 2005.
- [4] Abu Sayed Rahaman, Md. Imdadul Islam and M. R. Amin, "Performance Evaluation of Two-Hop Wireless Link Under Rayleigh and Nakagami-m Fading Channel", *International Journal of Engineering and Technology (IJET)*, (Singapore), vol. 3, no. 5, pp. 454-459, 2011
- [5] Y. Yang and G. M. Masson, "Nonblocking broadcast switching networks", *IEEE Trans. Comput.*, vol. 40, pp. 1005-1015, Sep. 1991.
- [6] Asfara R. Rafiq, N.A. Siddiky, Md. Imdadul Islam, M.R.Amin," Optimization of k- fold wireless network using M/M/n/n+q Traffic Model" ICECE 2008, Buet, Dhaka, Paper ID ICECE 125 2008.
- [7] MM Hasan, Most Tajmary Mahfuz, Woakil Uddin Ahamed, Syed Akhter Hossain, " Requirement Analysis for Context-Awareness to Create a Digital Persona in eHealth based Pervasive Intelligent Environment", *Journal of Information and Communication Technologies (ISSN 2047-3168)*, vol. 2, issue 3, pp.11-15, United Kingdom, 2012.
- [8] P. Feldman, J. Friedman, and N. Pippenger, "Wide-sense nonblocking networks", *SIAM J. Discr. Math.*, vol. 1, no. 2, pp. 158-173, May 1988.
- [9] N. McKeown, A. Mekkittijul, V. Anantharam, and J. Walrand, "Achieving 100% throughput in an input-queued switch", *IEEE Trans. Commun.* , vol. 47, pp. 1260-1267, Oct. 1999.
- [10] M. Andrews, S. Khanna, and K. Kumaran, "Integrated scheduling of unicast and multicast traffic in an input-queued switch", in *Proc. IEEE INFOCOM*, 1999, pp. 1144-1151.
- [11] Y. Yang and J. Wang, "On blocking probability of multicast networks", *IEEE Trans. Commun.*, vol. 46, pp. 957-968, Jul. 1998.
- [12] Y. Yang and J. Wang, "Nonblocking k-fold multicast networks", *IEEE Trans. Parallel Distrib. Syst.*, vol. 14, pp. 131-141, Feb. 2003.

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